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Elastic anisotropy of bone and dentitional tissues

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The calculation of the scalar compressive and shear anisotropy factors developed for single crystal refractory compounds has been adapted to the anisotropic elastic stiffness coefficients determined by a number of ultrasonic measurements of bone based on transverse isotropic symmetry. Later, this work was extended to include the ultrasonic measurements of bone based on orthotropic symmetry. Recently, the five transverse isotropic elastic constants for both wet and dry human dentin were determined using resonant ultrasound spectroscopy. The five transverse isotropic elastic constants for wet bovine enamel and dentin had been calculated based on modeling of ultrasonic wave propagation measurements and related data in the literature. The scalar compressive and shear anisotropy factors have been calculated from both these sets of data and are compared with a representative set from those published previously for both human and bovine bone and both fluoro- and hydroxyl-apatites.

1. Introduction

Comparison of the full sets of elastic stiffness, C_{ij} (or compliance, S_{ij}) coefficients for anisotropic materials, including biological tissues such as bone, by different authors or by different techniques or from different samples of the same material is difficult as the respective C_{ij} (or S_{ij}) usually do not scale one to another. Therefore, Katz and Meunier [1] adapted the calculations of Chung and Buessem [2] for refractory single crystals with hexagonal symmetry to bone, treating it as a textured material solid with transverse isotropic symmetry. In both cases, 5 independent C_{ij} (or S_{ij}) are required to fully characterize the anisotropic properties. Later, they extended this work to include measurements of bone as a textured material solid with orthotropic symmetry [3].

For a transverse isotropic solid the compressive anisotropy is given in terms of compliance by:

$$Ac^* = (S_{33} + 2S_{13})/(S_{11} + S_{12} + S_{13}), \qquad (1)$$

i.e., the ratio of the linear compressibility along the unique axis, denoted by 3, to that perpendicular to the unique axis, i.e., the 1, 2 plane. Ac^* also may be written in terms of stiffnesses by:

$$Ac^* = (C_{11} + C_{12} - 2C_{13})/(C_{33} - C_{13}). \qquad (2)$$

Similarly the scalar shear anisotropy factor, As^*, the ratio of unique axis shear moduli to the in-plane value, in terms of the elastic stiffness is given by:

$$As^* = C_{44}/C_{66}. \qquad (3)$$

where $C_{66} = 1/2(C_{11} - C_{12})$.

However, both these ratios, given by Equations 2 and 3, can vary over a wide range, again inhibiting direct comparisons between sets of data. Fortunately, differences between the Voigt (upper bound) and Reuss (lower bound) moduli also are measures of both the compressive and shear elastic anisotropy [4]. Therefore, Chung and Buessem [2] used the following equations, defining both the compressive and shear anisotropy respectively, for systems with either cubic or hexagonal symmetry:

$$\begin{aligned} Ac^* &= (K^{\mathrm{v}} - K_{\mathrm{R}})/(K^{\mathrm{v}} + K_{\mathrm{R}}) \\ As^* &= (G^{\mathrm{v}} - G_{\mathrm{R}})/(G^{\mathrm{v}} + G_{\mathrm{R}}) \end{aligned} \qquad (4)$$

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TABLE I Elastic constants for both wet and dry human dentin samples (based on a transverse isotropy model); and wet bovine dentin and enamel (based on computational modeling)

Condition	C_{11}	C_{33}	C_{12}	C_{13}	C_{44}	$C_{66}[= 1/2\,(C_{11}–C_{12})]$
Human dry	36.7	36.5	14.7	15.1	11.1	11.0
Human wet	42.6	34.6	25.4	19.7	9.40	8.60
Bovine dentin[a]	37.0	39.0	16.6	8.70	5.70	10.2
Bovine enamel[a]	115.	125.	42.4	30.0	22.8	36.3

[a]Model calculation based on Katz and Ukraincik [7].
All values in GPa.

where K is the bulk modulus and G is the shear modules. Thus, all the anisotropy factors are expressed by values between 0 and 1 making comparisons straight forward. An even more useful description of the scalar factors is to present them as percentages, i.e.:

$$\begin{aligned} Ac^*(\%) &= 100\,(K^{\text{v}} - K_{\text{R}})/(K^{\text{v}} + K_{\text{R}}) \\ As^*(\%) &= 100\,(G^{\text{v}} - G_{\text{R}})/(G^{\text{v}} + G_{\text{R}}) \end{aligned} \tag{5}$$

It is clear that the greater the value of Ac^* (%) or As^* (%), the greater the degree of anisotropy. Thus direct comparison can now be made between different measurements on different specimens in different investigations. Indeed, it was shown by Katz and Meunier [1, 3] that these anisotropy factors also could be used as an indicator of whether an experiment involved artifacts of measurement, or modeling of properties, or was from pathologic samples, as one or more of these scalar factors would deviate considerably from the small range of values found for a large number of different samples by different investigators using different measuring techniques.

2. Results

Recently, Kinney *et al.* [5] used the technique of resonant ultrasound spectroscopy (RUS) to measure the elastic constants (C_{ij}) from both wet and dry human dentin samples. Their data showed that the samples exhibited transverse isotropic symmetry requiring only 5 independent elastic constants to fully characterize the material, Table I (adapted from Table I in Kinney *et al.* [5].

In addition, reviewing the literature on anisotropy in calcified tissues uncovered the ultrasonic wave propagation study by Lees and Rollins [6]. In this paper they modeled both bovine dentin and enamel as having transverse isotropic symmetry and derived the 5 independent elastic constants for such symmetry based on their limited ultrasonic wave propagation measurements on wet specimens and the modeling by Katz and Ukraincik [7]. These data are also given in Table I.

Using the equations for K^{v}, K_{R}, G^{v} and G_{R} (given in the Appendix of both Katz and Meunier papers [1, 3]), $Ac^*(\%)$ and As^* (%) have been calculated for the data in Table I. These are all displayed in Table II.

The values of $Ac^*(\%)$ and As^* (%) from Table II, are compared in Table III with some representative values of both human and bovine bone as calculated previously [1, 3].

Fig. 1 is the graph of As^* (%) versus $Ac^*(\%)$ for the data in Table III.

TABLE II Values of K^{v}, K_{R}, G^{v}, G_{R}, $Ac^*(\%)$ and As^* (%) for both wet and dry human dentin and wet bovine dentin and enamel (Voigt and Reuss values are in GPa)

Condition	K^{v}	K_{R}	G^{v}	G_{R}	$Ac^*(\%)$	As^* (%)
Dentin dry [5]	22.19	22.19	10.97	10.97	0.0062	0.0110
Dentin wet [5]	27.71	27.00	9.147	9.078	1.305	0.3770
Bovine dentin [6]	20.11	20.03	9.587	8.208	0.1990	7.750
Bovine enamel [6]	62.20	62.20	33.22	30.55	0.0040	4.195

TABLE III As^* (%) and $Ac^*(\%)$ for various types of hard tissues and apatites

Specimen type	$Ac^*(\%)$	As^* (%)
Human dentin dry [5]	0.006	0.011
Human dentin wet [5]	1.305	0.377
Bovine dentin [6]	0.199	7.750
Bovine enamel [6]	0.004	4.195
Bovine femur [8]	1.522	2.075
HAp [7, 9]	0.995	0.686
FAp [7, 9]	0.867	0.630
Bovine femur dried [10, 11]	1.391	0.981
Bovine femur [12]	2.627	5.554
Human femur dried [13]	1.036	1.055
Haversian [14]	1.080	0.775
Human femur [15]	1.504	1.884

As indicted above, different samples of the same bone generally will have considerable differences from one another in their sets of C_{ij}. This clearly is seen in the experiment by Van Buskirk and Ashman [15] in which they used ultrasonic wave propagation to obtain the full sets of 9 C_{ij} for a human femur at each of the four aspects around the periphery, Anterior, Medial, Posterior, Lateral, at fractional proximal levels, Z/L, of the femur, where Z is the level along the length of the femur, L; this was done at 5 equally spaced levels within the range of $Z/L = 0.3–0.7$. As^* (%) and $Ac^*(\%)$ have been calculated for both transverse isotropic and orthotropic symmetry from these data. The graph of As^* (%) versus Z/L, based on the transverse isotropy calculation, is given in Fig. 2; similarly, the graph of $Ac^*(\%)$ vs. Z/L, based on the transverse isotropy calculation, is given in Fig. 3.

3. Discussion

The C_{ij} for dry human dentin are most interesting. Clearly, the values of $C_{11} \approx C_{33}$ (average value 36.6 GPa) and $C_{12} \approx C_{13}$ (average value 14.9 GPa) along with the value of 11.1 GPa for C_{44}, observed in Table I, would imply an even higher symmetry than transverse isotropy, i.e. there now would be only the

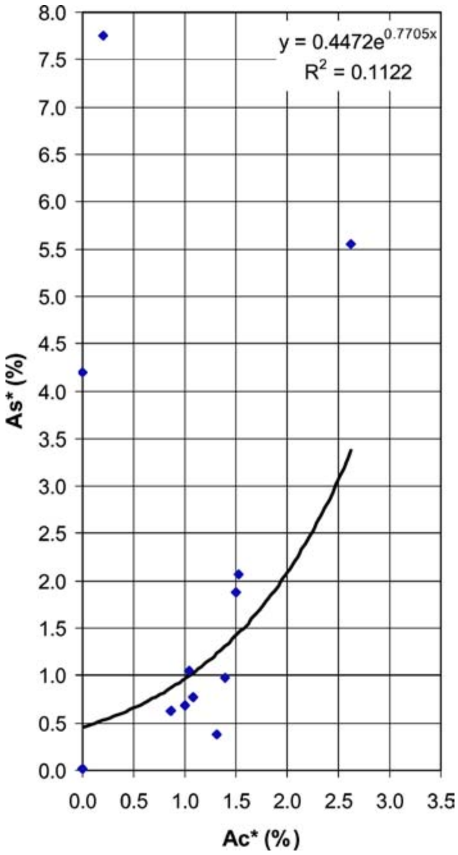

Figure 1 As^* (%) versus $Ac^*(\%)$ for various hard tissues and apatites. The data points are the values of $Ac^*(\%)$ and Ac*(%) tabulated on Table III as calculated from sets of the 5 transverse isotropic C_{ij} presented in the respective cited literature.

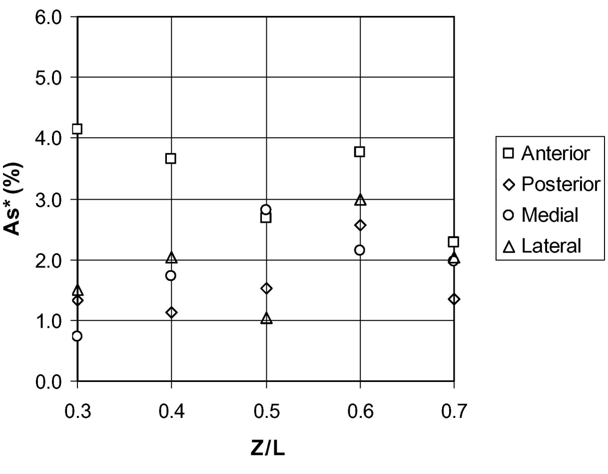

Figure 2 The graph of As^* (%) versus Z/L, for a human femur. The data points are the values of As^* (%) as calculated from sets of the C_{ij} measured by Van Buskirk and Ashman [15] for a human femur at each of the four aspects around the periphery, Anterior, Medial, Posterior, Lateral, at fractional proximal levels, Z/L, of the femur, where Z is the level along the length of the femur, L; this was done at 5 equally spaced levels within the range of $Z/L = 0.3$–0.7 using ultrasonic wave propagation; the data points presented are those based on assuming transverse isotropy.

three independent elastic constants necessary to describe cubic symmetry. Chung and Buessem state [2], "For a cubic material, the degree of elastic anisotropy has been defined as:

$$A^*_{\text{cubic}} = 3(A-1)^2/[3(A-1)^2 + 25A] \tag{6}$$

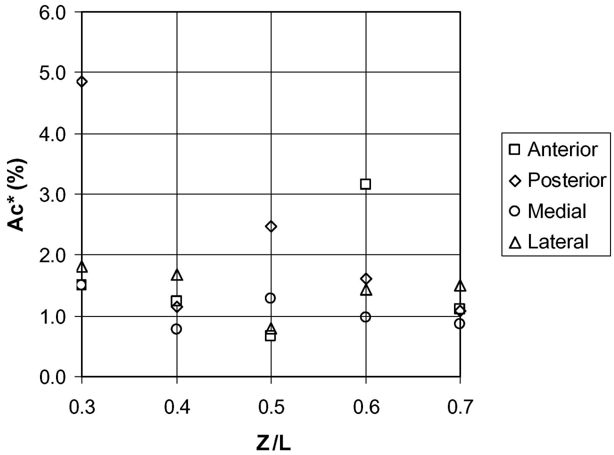

Figure 3 The graph of $Ac^*(\%)$ versus Z/L, for a human femur. The data points are the values of $Ac^*(\%)$ as calculated from sets of the C_{ij} measured by Van Buskirk and Ashman [15] for a human femur at each of the four aspects around the periphery, Anterior, Medial, Posterior, Lateral, at fractional proximal levels, Z/L, of the femur, where Z is the level along the length of the femur, L; this was done at 5 equally spaced levels within the range of $Z/L = 0.3$–0.7 using ultrasonic wave propagation; the data points presented are those based on assuming transverse isotropy.

Here, A, the usual cubic anisotropy factor, is given by:

$$A = 2C_{44}/(C_{11} - C_{12}) \tag{7}$$

(a) $A^* > 0$ regardless whether $A > 1$ or $A < 1$

(b) $A^* = 0$ for elastically isotropic materials, i.e. $A = 1$

Using the average values for C_{11} and C_{12} and the value for C_{44} cited above, $A = 1.023$, yielding a value of $A^* = 0.00006200$, or $A^*(\%) = 0.00620$, Table III. This value for $A^* \approx 0$ suggests that dry human dentin is very nearly elastically isotropic rather than even exhibiting the cubic symmetry inferred above. $A^*(\%)$ vs. A falls almost exactly at the minimum on the, "Graphical representation of the elastic anisotropy for cubic crystals", Fig. 1, Page 218 [2] supporting this suggestion of elastic isotropy. This isotropic-like behavior of the dry dentin may have clinical significance in patients with decreased saliva output [16, 17], or "dry mouth" syndrome in which there is an associated drying of tooth structure. There is independent experimental evidence to support this calculation of isotropy based on ultrasonic data. Small angle X-ray diffraction of human dentin yielded results implying isotropy in the collagen near the pulp and mild anisotropy in mid-dentin [18]. As the apatitic mineralites' orientations are in register with the collagen fiber orientations this implication supports the above calculation of isotropy for the dry dentin.

Note that in Fig. 2, the Anterior aspect of the femur, that is in tension during loading, has values of As^* (%) in some positions considerably higher than those of the other aspects, whereas in Fig. 3 it is the Posterior aspect, that is in compression during loading, that has values of $Ac^*(\%)$ in some positions considerably higher than those of the other aspects. Both graphs are based on

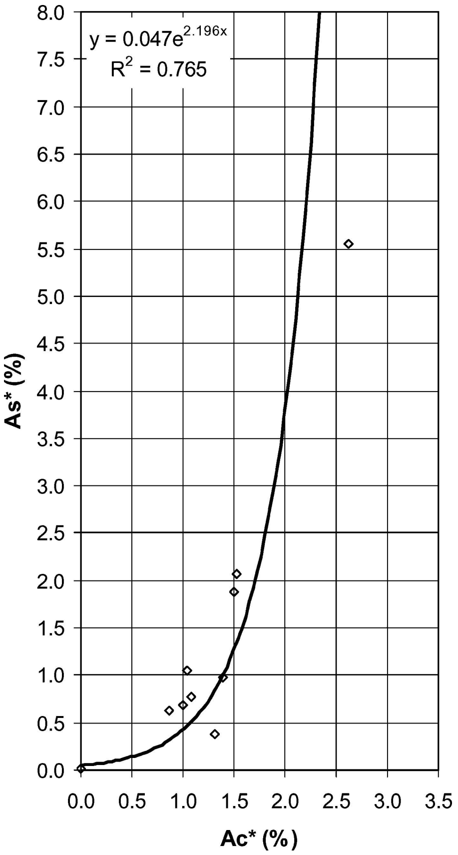

Figure 4 As^* (%) versus Ac^*(%) for various hard tissues and apatites. The data points are the same values of As^* (%) and Ac^*(%) tabulated on Table III as calculated from sets of the 5 transverse isotropic C_{ij} presented in the respective cited literature with the exception that the clearly anomalous data points for bovine dentin [6] and enamel [6] have been removed.

the transverse isotropic symmetry calculations. However, the identical trends were obtained based on the orthotropic symmetry calculations. It is clear that in addition to the moduli varying along the length and over all four aspects of the femur, the anisotropy varies as well, reflecting the response of the femur to the manner of loading during its function.

As pointed out earlier [1, 3], deviations from the relatively restricted area enclosing the plot of As^* vs. Ac^* could be used to point out anomalous results whether due to measurement artifacts or limitations in modeling the symmetry as transverse isotropic. This is especially true for measurements made by mechanical testing where deviations in orientation from the principal axes may occur due to the sizes of differently oriented specimens from different regions of the bone, e.g. see the calculations for Knets [19] in comparison with ultrasonically derived values in Table I in Katz and Meunier [1]. Such anomalous behavior is clearly the case with the data based on the modeling for wet bovine dentin and enamel [6], see Fig. 1. Note that in both cases As^* (%) and Ac^*(%) values result in locations on Fig. 1 that differ significantly from the confined area encompassing all the other ultrasonic measurements. The reason for this anomalous behavior may be due to the combination of not measuring the 5 elastic constants independently of either dentin or enamel, and that the model used to calculate the full set of C_{ij} for each [7] involved orientations for which the symmetry may have been reversed. Fig. 4 is the same graph as Fig. 1, As^* (%) versus Ac^*(%), however, with the outlying data points for the bovine dentin and enamel omitted; this yields a significant improvement in R^2.

It is clear from the above discussion, that these scalar anisotropy factors provide significant information above and beyond that provided by modulus measurements alone, especially when the anisotropic elastic properties are important in understanding the structure/property/function relationships in materials.

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